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A comparison of two approaches to place-value instruction in first-grade classrooms

Barbara Strahl Moore
San Jose State University

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Moore, Barbara Strahl, M.A.

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U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

A COMPARISON OF TWO APPROACHES
TO PLACE-VALUE INSTRUCTION
IN FIRST-GRADE CLASSROOMS

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Barbara Strahl Moore

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APPROVED FOR THE COLLEGE OF EDUCATION

Lynne E. Gray

Dr. Lynne Gray

Irene Miura

Dr. Irene Miura

M. Stephanie Steffey

Dr. M. Stephanie Steffey

APPROVED FOR THE UNIVERSITY

Serena W. Stanford

ABSTRACT

A COMPARISON OF TWO APPROACHES TO PLACE-VALUE INSTRUCTION IN FIRST-GRADE CLASSROOMS

by Barbara Strahl Moore

The teaching of place value continues to be a significant problem for U.S. educators. In order to add to current knowledge concerning the acquisition of place-value concepts, two methods of place-value instruction were compared. Fifty-two first-grade students served as subjects for the study. One group received place-value instruction using a standard textbook with concrete manipulative materials as an occasional supplement. The other group received place-value instruction through place-value recording strips using Base 10 blocks as an integral part of the learning experience. It was hypothesized that the first-grade students who received place-value instruction via the recording strip approach would achieve higher scores in place-value understanding than first-grade students who received place-value instruction through a standard textbook approach. Results indicated that there were no significance differences between the two groups.

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CHAPTER I

Introduction

The Problem

The concepts of place value, the meanings attached to particular digits in a multidigit numeral, are generally introduced in the first-grade mathematics curriculum. Recent popular mathematics textbooks published by Harcourt, Brace, Jovanovich (1991), Houghton Mifflin (1991), MacMillan/McGraw-Hill (1991), Scott, Foresman (1991), and Silver Burdett & Ginn (1991) all follow this pattern. The teaching of place-value concepts in first grade, however, continues to be a significant problem for mathematics teachers in the United States (Lindquist, 1989). According to Labinowicz (1985), the problem is that the "introduction of the place-value notational system in the first grade assumes that these children are already thinking in terms of systematic groupings by tens" (p. 280) when, in fact, most are not.

The problems children have in understanding place value may arise from a number of different causes. Recent research (Fuson, 1981; Fuson & Briars, 1990; Miura & Okamoto, 1989) suggests that the method by which the concepts of place value are taught may be particularly significant. Other researchers (Ross, 1986) theorize that first-grade students have not yet reached the appropriate developmental level which would enable them to understand place value. Still others (Baroody, 1989; Resnick, 1983) conclude that first graders are not likely to have had the necessary prior experience with counting which provides the foundation for understanding place-value concepts.

Place-value understanding involves knowing the meaning of individual positions in a multidigit numeral, as well as the value of a particular digit in different positions in a number. It is too often the case, however, that children are able to correctly complete textbook exercises on place value without learning the underlying meanings of place value. Recent findings (Fuson & Briars, 1990) from place-value research indicate that the introduction of place-value concepts in first-grade texts may cause children to resort to rote procedures in order to complete place-value tasks without allowing them time to construct the necessary understandings about underlying meanings.

In their report on the fourth mathematics assessment of the National Assessment of Educational Progress (NAEP), Koubka, Brown, Carpenter, Lindquist, Silver, and Swafford, (1988) noted that even at the age of nine or ten, "approximately two-thirds of the third grade students appeared to have mastered place value involving tens, but fewer than half were successful with tasks involving place-value notions beyond tens" (p. 65). Research by Ross (1986) has demonstrated that young children's understanding of place value can be superficial; that is, they may be able to do exercises related to place value and even correctly execute the standard addition and subtraction algorithms, but they do not understand what they are doing. It is not until fourth grade, Ross concluded, that most children understand the underlying meanings of place value.

When teaching place value, most first-grade teachers follow the district's curriculum guidelines and approved textbook. The majority of currently available texts almost exclusively employ drill and practice exercises that rely on line drawings and repeated writing in workbooks of the numerals from 1 to 100 to teach concepts. Manipulatives, which are concrete materials used to facilitate the teaching of mathematical concepts, are often pictured and are sometimes recommended as supplemental aids, but are not required for completing the regular activities. Few teachers find time for supplemental activities, and even fewer use a manipulative approach as their primary method of teaching.

According to Fuson (1981), "there are some things that are easier for children to understand than to do and others that are easier simply to do than to understand" (p. 62). Doing without understanding, however, rarely results in knowledge that is useful. Problems that arise in the development of understanding of place value, then, involve both doing and understanding and are usually related to one or more of the following three areas:

1. The English system of number words. There is potential for confusion because the English language does not mirror the base ten system exactly. For example, the numeral two when joined with a ten becomes a completely new number word called "twelve," instead of something like "ten two" (Fuson & Briars, 1990).

2. The position of a digit in the numeral. When the position of a digit changes, the value of a number changes. For example, in the number 12, the 2 means two units, and the 1 means one ten. When the digits are reversed to make the number 21, the 1 now represents one unit, and the 2 represents two tens. According to Fuson (1990), the concept students must know in order to understand place value in the base ten system is that "each position implicitly takes a value ten times larger than the value of the position on its right" (p. 275).

3. The knowledge of written symbols for number words. A problem arises when we change number words into written symbols, for example, writing 12 for twelve and 25 for twenty-five. In order to make the connections necessary for understanding place value, students need to know the symbols for number words, and they need to participate in concrete experiences which use number words in meaningful ways (Baroody, 1989). For place-value knowledge to be acquired,

students need to begin to use the number words concretely, in a variety of counting, building, and writing experiences (Resnick, 1983).

The Teaching of Place Value

Understanding place value involves knowing the meanings of individual positions in a number, as well as the value of a particular digit in a multidigit numeral. In order to attain this kind of place-value understanding, a teaching strategy is needed which can facilitate children's learning of the underlying concepts. Such a strategy would avoid activities in which children are rotely performing exercises without building understandings. The current textbook model teaches place value through a series of paper and pencil tasks in which students spend a great deal of time mechanically partitioning two-digit numbers into groups of ones and tens. This teaching strategy does not allow children to have experience manipulating concrete materials and does not seem to be effective for the majority of students (Lindquist, 1989).

Some first-grade teachers have begun to experiment with a recording strip strategy which involves building numbers with Base 10 blocks and then writing the corresponding numbers on strips of paper. This strategy uses manipulatives as an integral part of instruction. A review of the available literature indicates that the effectiveness of recording strip strategies has not been thoroughly compared to other strategies for place-value instruction. Findings from such comparative studies would help to determine which instructional strategy best facilitates the development of higher level understandings of place-value concepts.

Purpose of the Study

It was the purpose of this study to compare two strategies for place-value instruction in the first grade in order to determine which was more effective in developing understanding of place-value concepts. One strategy used a standard approach based on textbooks and the District's Curriculum Guide with manipulative materials included irregularly as a supplement. The second strategy used place-value recording strips with Base 10 block manipulatives as an integral part of the instruction. In comparing these two strategies for place-value instruction in first grade, the research attempted to add to current knowledge concerning the acquisition of place-value concepts.

Hypotheses

1. First-grade students who receive place-value instruction via the recording strip approach will achieve higher scores in place-value understanding than first-grade students who receive place-value instruction through the standard textbook approach.

2. First-grade students receiving place-value

instruction via either method will not differ on performance of standard addition algorithms because such performance is a mechanical skill at this level and, therefore, is not influenced by better understanding of place-value concepts.

CHAPTER II

Review of the Literature

The teaching of place value continues to be a significant problem for U.S. educators (Lindquist, 1989). Place-value, the meaning assigned to a digit in a number, is generally introduced in the first-grade mathematics curriculum. Place-value understanding involves knowing the face value of individual digits, as well as the value of the digit in different positions in a given number. Children must learn the underlying meaning of place-value to succeed in elementary arithmetic; however, such understandings have not been adequately developed by typical textbook place-value exercises. Recent research (Koubka et al., 1988; Ross, 1986) has suggested that first grade may not be the appropriate time to introduce the concept because it does not seem to be fully understood by most students until the third or fourth grade. Other research (Fuson, 1981; Fuson & Briars, 1990), however, has suggested that developmental level may not be the problem. These studies focus on inadequacies in the methods used to teach the concepts of place value.

Most first-grade teachers use an approach to teaching place value which follows the textbook adopted by their district. The textbook strategy typically employs drill and practice in writing numbers and includes manipulatives only as a supplement to instruction. Some first-grade teachers use newer strategies which feature manipulatives, and some are experimenting with a technique that uses Base 10 blocks and recording strips as the primary means of instruction. Base 10 blocks are wooden blocks which represent the base ten numeration system. The blocks are divided into "unit" blocks, which represent ones; "ten" blocks (or "longs"), which are equal to ten ones; "hundred" blocks (or "flats"), which are 100 unit blocks in a ten by ten square (or ten longs); and "thousand" blocks, which are ten by ten by ten cubes. Recording strips are long, thin strips of paper which are used to record consecutively the number and kind of Base 10 blocks used to represent each counting number. The number of blocks is recorded in a labeled box which corresponds to a place value: that is, thousands, hundreds, tens, and ones.

Definition of Place Value

According to Labinowicz (1985), conventional place-value notation is a system of representing quantity through symbols. Inherent in place-value notation for the base ten numeration system is a systematized grouping by tens. The grouping enables us to count large collections of objects.

Numbers can be represented in three ways: (1) concretely (with Base 10 blocks, for example); (2) pictorially (using drawings of objects equal to a specified number); or (3) symbolically (using digits alone without blocks or pictures). As we move farther away from the concrete reality, it is necessary to "read" meaning into the symbols.

Miura and Okamoto (1989) state that "place value is the meaning of tens and ones in a two-digit number" (p. 109). They studied the cognitive factors which affect children's abilities to make mental representations of number, as well as the role that language plays in the development of these abilities. A child who has the ability to distinguish the tens part and the ones part possesses the necessary prerequisites to understand place value.

Importance of Place Value

In the recent publication, *Curriculum and evaluation standards for school mathematics* (National Council of Teachers of Mathematics, 1989), it is stated that, "Understanding place value is a critical step in the development of children's comprehension of number concepts" (p. 39). Because number concepts are an important part of our lives, children must learn how to use them effectively. Place-value knowledge enables students to make sense of number concepts and to understand the underlying meanings of computation rules. Students with good place-value understanding would, specifically, understand the value of a digit in a numeral and be able to flexibly represent numbers in different ways.

Burns and Tank (1988) are other authors who stress the importance of place value. They assert that place-value knowledge is particularly important in understanding regrouping, gaining number sense, and making mental computations. In *The mathematics framework for California public schools* (California State Department of Education, 1985), readers are reminded that place-value knowledge is the basis for all work in arithmetic, and that understanding of place value enables students to progress in addition, subtraction, multiplication, and division. Without this fundamental understanding, mathematics would be impossibly difficult for students.

Children's Understanding of Place Value

The California framework (cited above) focuses on the distinction between performance (that may be based on the rote memorization of rules and procedures) and understanding. Although the authors of the framework do not advocate eliminating the teaching of rules and procedures, they do advocate using them only in meaningful, concrete contexts. The goal for place-value instruction, according to these authors, should be to teach for understanding as well as performance.

The *Mathematics model curriculum guide, kindergarten through grade eight* (California State Department of Education, 1987) reiterates the framework's goal of developing essential understandings in children. "Essential understandings are a basic set of ideas which grow more complex over time" (p. 15). Specific to place-value understanding is the concept of grouping by tens, which involves knowing the multiples of ten, and knowing the value of a digit in a numeral. These understandings might be accomplished through the building of number representations using concrete materials, as well as through experiences which call for reading and writing numbers for useful purposes like finding patterns in counting sequences.

The distinction between rote number manipulations and understanding has also been discussed by Burns and Tank (1988) who assert that "even though children may appear to know the value of a number, they do not know the underlying meaning" (p. 62). This observation is the crux of the problem for place-value instruction.

Ross (1986) designed a study to determine what developmental factors affect children's understanding of place value. For the purposes of the study, she limited the definition of place value to the ability to partition a whole number into a tens part and a ones part. Sixty children between grades two and five were included in the study. The students were presented with 18 place-value tasks. The results of the children's performance on these tasks provided Ross with information which enabled her to propose a five-stage developmental model of children's acquisition of place-value knowledge. Children in Stage I can read a two-digit number, but do not attribute meaning to the individual digits in a number. Stage II children have limited knowledge of the positions of digits in a number, yet do not understand the quantity of each digit. Stage III children "interpret digits by their face values in such a way that they sum to the whole" (p. 2); that is, they think the 2 in 25 stands for two of one kind of object and the 5 represents five of a different kind of object. Stage IV children can interpret the tens digit as representing groups of ten, yet their understanding and performance are limited and unreliable. Stage V children show a reliable knowledge of place value because they are able to separate a two-digit number into tens and ones. Using this model as a guide, Ross found that it was not until fourth grade that most children reach Stage V. Ross concluded, "The levels of cognitive development typical of children in the primary grades limit their ability to understand place value" (p. 45).

Research on Traditional Text-Oriented Approaches

Work by Romberg and Carpenter (1986) on traditional text-oriented instruction of place value suggests that it is based on the Behaviorist theories of psychologists such as B. F. Skinner and E. Thorndike. These theories advocate the breaking down of concepts into small, discrete chunks and teaching them in short steps. Romberg and Carpenter concluded that, "Thorndike's theories, which focused on skill development and practice, [have] significantly influenced the design of the elementary arithmetic curriculum" (p. 852). Critics of Behaviorist learning theories claim that such instruction is too fragmented and that it divides the curriculum into too many separate parts which are never re-integrated in children's minds.

Resnick (1983) and Howson (1983) are two other authors who concur with Romberg and Carpenter's conclusions. According to Resnick (1983), typical textbook teaching of place value is based on Behaviorist theory which "concentrates on number performance skills and views growth in mathematical ability as the addition of successive performance skills" (p. 10). Howson (1983) wrote that the Behaviorist movement focused on observable behaviors like "can add two, two-digit numbers" to the exclusion of issues of understanding. Teachers are instructed to (a) specify precise behavioral objectives, (b) sequence them in order of difficulty, and (c) provide extensive opportunities for practice and reinforcement.

Research by Baroody (1990), Fuson (1990), and Fuson and Briars (1990) examined textbook practices in place-value instruction. Fuson studied five aspects of standard textbook approaches to the teaching of place-value concepts and multidigit addition and subtraction. The first, second, fourth, and fifth aspects dealt primarily with multidigit addition and subtraction. The third dealt specifically with place value. She found that textbooks only focused on place-value exercises before multidigit addition and subtraction concepts were introduced. Fuson hypothesized that these things need to be considered simultaneously and concluded that "Understanding of place value is multi-faceted and prolonged and accompanies and follows understanding of multidigit addition and subtraction" (p. 274).

Fuson and Briars (1990) designed a study to test this hypothesis. The study included 169 first- and second-grade students representing eight different classes in a Chicago suburb. The participating students, who were grouped by mathematics achievement, received mathematics instruction through a learning/teaching approach developed by Fuson and Briars that featured the use of Base 10 blocks. Students' performance on pre- and posttests was compared to determine whether their understanding of multidigit addition and subtraction would be enhanced if taught concurrently with place-value instruction. Teachers were given training in this alternative teaching method, and each class received a set of Base 10 blocks.

The first phase of instruction was to teach the children "counting on" procedures in order to help them find sums and differences to 18. "Counting on" is a strategy which children can use for addition which involves continuing to count from the first addend instead of going back to one and starting the entire count over again. Exploration of the Base 10 blocks followed, allowing time for children to learn the descriptive names for the blocks. Through trading, children learned to exchange ten ones blocks for one tens block, ten tens blocks for one hundreds block, and ten hundreds blocks for one thousand cube. Numeral cards were also introduced so that children could see the written forms of the numbers. Addition of up to four-digit numbers was shown using calculating sheets, digit cards, and worksheets.

As children became competent computing with the Base 10 blocks, they could, after being checked by an adult, do the problems without the blocks. The time frame for the beginning exploration and addition section was from three to six weeks. Two to four weeks were allocated for the subtraction unit.

To measure the skills and understanding achieved, two kinds of tests were given: addition and subtraction calculation tests, and place-value and multidigit addition tests. These same tests served as both the pretest and posttest measures.

Fuson and Briars (1990) concluded that there was significant improvement in understanding place-value concepts and in performing multidigit addition and subtraction problems for students who studied these things simultaneously and who used Base 10 blocks. As a result of this approach, the second-grade students who were given place-value tasks adapted from Ross (1990) "showed performance considerably above that ordinarily shown by second graders receiving traditional instruction" (Fuson & Briars, 1990, p. 196). The final conclusion from this study was that an approach which features the use of Base 10 blocks is an effective method "if the focus of such learning is understanding and not just procedural competence" (p. 204).

Research on Alternative Approaches to Place-Value Instruction

Baroody (1990) reviewed the research on place-value instruction and concluded that there is a need for change and improvement on the current textbook approach. He advocated using the semi-integrated approach he developed, which is described in detail in his 1987 book, *Children's mathematical thinking*. Baroody (1990) stated that his method (unlike the textbook method which assumes that multidigit concepts can be learned quickly and passively) calls for the children's active involvement in "meaningful models [which are linked to] written symbols" (p. 282). In looking at the textbook treatment of place value, Baroody concluded that "the existing treatments of place value do not adequately help children construct a multidigit concept" (p. 281) because the concepts are not introduced meaningfully. Baroody discussed the possible benefits of

Base 10 blocks in learning place-value concepts. He suggested there may be a need for these concrete materials to help illustrate place-value concepts. He further stated that there is a need for more research to investigate the specific issues of how to change place-value instruction and when it should be presented in the curriculum.

Building on the research from her 1986 study, Ross (1990) studied the effect of instruction on children's place-value understanding. Her 1990 study investigated the textbook method of instruction as well as a manipulative approach using Base 10 blocks. She hypothesized that the children who received instruction using manipulatives would perform better on place-value tasks that reflected a more complete understanding of place value.

Children were divided into two sample groups. Sample One contained the 60 children involved in the 1986 study. These children were chosen randomly to represent varying school instruction, school size, and the schools' socio-economic class. They received place-value instruction through various textbook series.

The 40 children in Sample Two had a solid foundation in manipulative math experiences delivered by teachers who used one of three resource books: Jenkins and Nordberg (1977), *Place-Value and Regrouping Games*; Neufeld and Lucas (1976), *Number-Blox, Books A and B*; or Laycock (1977), *Base Ten Mathematics*. Ten children from each grade level, second through fifth, who had attended the same school for at least three years, were randomly selected and interviewed. Ten tasks involving number representation, digit correspondence, positional knowledge, and class inclusion were given to the students individually in both sample groups at both the pretest and the posttest.

On the basis of the results of this study, Ross (1990) concluded "that neither instruction nor cognitive development alone account for the variation . . . in children's understanding of place value" (p. 13). The most important determinant seemed to be the interaction between these two factors. Ross further concluded that another important factor to consider in assessing children's place-value understanding is the measurement tool used. She recommended student interviews be used to document improvement made following appropriate place-value instruction. She found that an interview process, similar to the one used by Miura and Okamoto (1989) was a useful way to analyze children's thinking.

Other Related Work

Research by Miura and Okamoto (1989) suggests that language may be an important consideration in the development of place-value understanding. They sought to determine the role which language played in the development of mental representations of numbers among first-grade English-speaking students and Japanese-speaking students.

They hypothesized that Japanese-speaking students would show a better understanding of place value because, in their language, number names reflect the base ten system. "The spoken numerals in Japanese correspond exactly to their written form" (p. 109), such as ten-one for 11. English-speaking students, on the other hand, may have more difficulty with cognitive representation of numbers because the English system of naming numbers is confusing. Children must memorize number names between 1 and 20 as well as decade names.

Miura and Okamoto (1989) designed a study to test these ideas using equal numbers of children in two different first-grade classes. The 24 English-speaking students attended a private school in an upper-middle-class area near San Francisco. The 24 Japanese-speaking students attended an upper-middle-class school in Tokyo. The U.S. children were interviewed in the first half of the school year and then again in the spring. The Japanese children were interviewed in the first half of the first-grade year. The method of instruction was held constant.

In order to assess the children's flexibility in mental representation of numbers, the children were presented individually with a set of five different numbers, one at a time, and asked to show each number using Base 10 blocks. Neither group had had experience with Base 10 blocks at the time of the testing. After completing the first construction, they were asked if they could do it again a different way. Each of the constructions was categorized using a system established by Ross (1986). The possibilities were:

1. A <u>one-to-one collection</u> where only unit blocks are used;

2. A <u>canonical base ten representation</u> where the minimum possible number of tens and units blocks is used; and

3. A <u>noncanonical base ten representation</u> where the student uses a combination of tens and units blocks, but more than nine unit blocks may be included in the construction.

In a second procedure, place-value understanding was assessed. Children were given five place-value tasks. Children were said to understand place value when they indicated they "understood the meaning of the individual digits in the numeral" (Miura & Okamoto, 1989, p. 111); for example, if they knew that the 2 in the number 28 meant 2 tens and the 8 in the number indicated 8 ones.

Japanese-speaking children used more canonical base ten representations than did the English-speaking children, and their answers to place-value tasks revealed a better understanding of place value. Miura and Okamoto (1989) concluded that language facilitated the ability to use base ten representations and made it easier for Japanese children to grasp the concepts of place value at an earlier level. English-speaking children, however, may need an intermediate step in their place-value instruction in which manipulatives such as Base 10 blocks would be used to build, count, and record numbers. Miura and Okamoto (1989) believe that "the use of manipulative materials (e.g., Base 10 blocks) to teach the counting sequence might promote better understanding of the structure of numbers" (pp. 113-114).

Support for providing early place-value experiences for young children can also be found in the *Mathematics model curriculum guide* (California State Department of Education, 1987) which recommends using manipulatives in the early grades to lay the appropriate foundation for place-value understanding. The *Curriculum and evaluation standards for school mathematics* (National Council of Teachers of Mathematics, 1989) also supports the early presentation of place-value concepts through a variety of procedures. According to Baroody (1990), "Children are able to make connections by relating various contexts to one another" (p. 283). Holmes (1985), as well as Baroody (1990), recommend using Base 10 blocks to give young children the opportunities to experiment with place value, especially with numbers of more than two digits.

Resnick (1989) has emphasized the need for children to have many experiences with counting of concrete materials such as Base 10 blocks and with informal methods of calculation which they developed themselves over time. "Counting methods of calculation provide a reliable way for children to generate answers for themselves to arithmetic problems they may encounter" (p. 177). Resnick writes that children need to build their knowledge of number concepts through inventing their own strategies. She states that:

> For most children, extensive practice in counting actual objects eventually leads to an ability to use the count words (one, two, three, etc.) themselves as objects to count, and they thus become able to engage in mental counting. This produces both efficiency and flexibility in solving addition and subtraction problems. It also gives evidence of considerable mathematical understanding. (p. 164)

A Strategy to Improve Children's Understanding of Place Value

Research on the use of Base 10 blocks (Baroody, 1990; Fuson, 1990; Fuson & Briars, 1990), research on building of base ten representations (Miura & Okamoto, 1989), and research on counting (Resnick, 1989) highlight these three areas as important components for developing place-value knowledge. An instructional strategy for introducing place-value concepts which takes into account all of these areas is the recording strip/manipulative based approach developed by Neufeld and Lucas (1976). This approach uses Base 10 blocks as an integral part of the instruction, uses counting activities, and focuses on number constructions. Neufeld and Lucas (1976) have found this approach to be effective in the development of place-value understanding because it enables children to progress from the concrete to the abstract by providing a "physical model of our base ten counting system" (p. iii). Children progress through levels of cognitive development starting with exploration of the Base 10 blocks, continuing with counting and trading of units blocks, and then recording the block constructions on recording strips according to their place value.

In their textbook, *Teaching elementary school mathematics for understanding*, Marks, Hiatt, and Neufeld (1985) stressed the need for primary-school children to be presented concepts through the use of concrete materials such as Base 10 blocks and recording strips. They explained:

> Place-value, a very subtle idea, can be understood by young children if adequate preliminary tasks are assigned. They must include manipulation of concrete representations of the base ten numeration system in early place-value work and their work must be done within the framework of counting. (p. 61)

Summary

Place-value instruction is generally introduced in the first-grade mathematics curriculum. Some recent researchers (Koubka et al., 1988; Ross, 1986) have suggested that this is not the appropriate grade level because the concept does not seem to be fully understood until third or fourth grade. Other researchers (Romberg & Carpenter, 1986) have pointed out that traditional text-oriented instruction in place value concentrates on low level objectives with the emphasis on algorithmic paper and pencil skills. Research by Baroody (1990), Fuson (1990), and Fuson and Briars (1990) supports the use of Base 10 blocks in learning place-value concepts. Miura and Okamoto (1989) also conclude that Base 10 blocks might provide an effective method for place-value instruction.

Ross (1990) has indicated the need for continued research into specific types of place-value instruction to determine which is most effective in facilitating understanding of the concepts. In reviewing the existing literature, it is evident that the most effective instructional strategy for teaching place-value concepts has not yet been agreed upon by a majority of researchers. The present study was designed to compare a traditional text-oriented approach with the recording strip approach to place-value instruction for first-grade students in order to determine which is more effective in developing understanding of the concepts. The purpose of this study, therefore, was to answer the following research question: Is there a difference in the level of understanding of place value in first-grade students who have received instruction using the recording strip approach with Base 10 blocks as compared to first-grade students who have received instruction through text-oriented approaches?

CHAPTER III

Methodology

The purpose of this study was to compare two approaches to place-value instruction in first grade. One approach used a standard textbook with manipulatives as an irregular supplement. The second approach used place-value recording strips and Base 10 blocks as the primary means of instruction.

The standard textbook approach is exemplified in the first-grade books of such widely used publishers as Harcourt, Brace, Jovanovich (1991); Houghton Mifflin (1991); MacMillan/McGraw-Hill (1991); Scott, Foresman (1991); and Silver Burdett & Ginn (1991). The concepts of place value are taught similarly in all five series through paper and pencil tasks in which students are asked to partition numerals into a tens part and a ones part. For example, students might be asked to circle groups of ten objects from a larger collection and then write the correct number of tens, or they might be asked to identify and write tens for numbers through 90. Use of manipulatives such as Base 10 blocks, connecting cubes, bean sticks, and counters is sometimes suggested by the publishers in their teachers' guides as a supplemental aid, but concrete materials are not required for completing the assigned tasks.

Base 10 blocks are wooden blocks that are often used to teach place value. The blocks provide a model of the base ten numeration system. They come in four sizes: unit cubes (or "ones") which represent units, tens blocks (or "longs") which are equivalent to ten ones in a single row, hundreds blocks (or "flats") which are equivalent to ten tens blocks or a ten by ten array of units, and thousands cubes which are equivalent to ten hundreds blocks or a ten by ten by ten cube. In the recording strip approach, children start by building the number one with the Base 10 blocks. As each unit block is counted, one at a time, the children write the corresponding numeral on the recording strip. They start in the ones column first. After nine blocks are counted, the children can start to record in the tens column. Building, counting, and recording can continue over time into the hundreds and thousands as Base 10 blocks are assembled and counted for each successive number. In following this building, counting, and recording procedure, children are able to see how the position of a digit affects its value. For the purpose of this paper, tens blocks will be represented by vertical lines (|) and units blocks by periods (.) so that 43 would be pictured here as ||||... and 29 as (||.........).

Subjects

Four first-grade classes participated in the study. Two classes served as the Experimental Group and two classes served as the Control Group. Four teachers were involved to avoid confounding treatment effects with teacher effects. The experimenter was not involved with the treatment, but served only to administer the pre- and posttests. The subjects were 52 first-grade students from low to middle socio-economic status (SES) families enrolled in a public school in Northern California. Demographic data are provided in Table 1.

Table 1

Demographic Data for Experimental and Control Groups

	Experimental N=26	Control N=26
Gender		
Male	62%	54%
Female	38%	46%
Ethnicity		
Caucasian	65%	69%
Hispanic	11%	11%
Black	11%	8%
Asian	4%	8%
Persian	4%	4%
East Indian	4%	0%
First Language		
English	85%	85%
Other	15%	15%

Although all students in the four classes were involved in the instruction, time constraints required that a randomly selected 13 students from each of the experimental and control classes be involved in the assessments. This resulted in a balanced 26 students in the Experimental Group and 26 students in the Control Group. The students in the Experimental Group (16 boys and 10 girls) were taught using the recording strip approach in which manipulatives, specifically Base 10 blocks, were used as an integral part of their instruction. The district's textbook was not used with the Experimental Group. The children in the Control Group (14 boys and 12 girls) received place-value instruction through standard paper and pencil tasks, based on the textbook, in which manipulatives were occasionally used as a supplement. Recording strips were not used at all in the Control Group. Base 10 blocks were used on at least one occasion by some of the children in the Control Group. Both the Experimental and the Control Groups occasionally used manipulative materials which were unstructured (straws and connecting cubes).

Pretest

The pretest, which was administered to each student individually by the researcher during the months of January and February, was adapted from the items used by Miura and Okamoto (1989) in their study of first-grade students. Prior to the pretest, none of the children had explored the Base 10 blocks. The pretest was divided into two parts, one testing the children's flexibility of number representation and the other testing the children's understanding of place value.

Part One. Part One of the procedure adapted from Miura and Okamoto was used to assess each child's cognitive representation of a number; that is, how number is understood by the child. Before the test began, the examiner gave a brief explanation and demonstration of the use of the Base 10 blocks to construct representations of numbers. Again, due to time constraints, children were given only three numbers to construct instead of the usual five. Each child was presented, in random order, with three cards (chosen from a collection of five containing the numbers 11, 13, 28, 30, and 42). For each card, the child was asked to read the number aloud and then to construct a representation of the number using the blocks. After the first representation was built (Trial 1), the child was asked if s/he could build the same number in a different way (Trial 2). For each of the three numbers, Trial 2 immediately followed Trial 1.

In order to describe the students' representations, a categorization system was adapted from research done by Ross (1986, 1990). There are three categories of correct responses:

1. A one-to-one collection--the use of only units blocks in a construction (for example, 28 unit blocks for the number 28);

2. A noncanonical base ten representation--tens blocks are used in conjunction with more than 9 unit blocks (for

example, one ten block and 18 units blocks for the number 28); and

3. A canonical base ten representation--the minimum number of tens and units blocks required for a correct representation (for example, the number 28 would be built using 2 tens blocks and 8 units blocks).

A subsequent scoring of the same data for the purpose of determining the depth and flexibility of students' understanding of number meaning was accomplished by assigning one point for each number that was correctly constructed across both Trial 1 and Trial 2. That is, if the student successfully built a number in two different ways, then one point was given. Each student could therefore attain a maximum score of three on Part One.

Part Two. The five questions used in Part Two of the assessment were designed to test place-value understanding and were also adapted from Miura and Okamoto (1989). For the purpose of this research, we assumed that a child understood place value only if s/he could consistently explain the function of each digit in a multidigit number. For example, when a child can say that the 3 in the number 32 represents 3 tens and the 2 represents 2 ones, s/he can be said to understand place value. The questions were administered and scored as follows:

Question 1. Each child was asked to look at a card with the number 32 on it. The child was asked to (1) point to the numeral in the ones position, (2) point to the numeral in the tens position, (3) use the blocks to represent the three, and (4) use the blocks to represent the two. Four possible total points could be obtained for this question, with each part worth one point.

Question 2. The examiner built the number 44 with the blocks (||||....) and then asked, "What number do the blocks make?" After the child responded, the card with the 44 was shown. The child was asked, "Which set of four blocks represents the four that is in the tens position?" This was followed by the question, "Which set of four blocks represents the four that is in the ones position?" Finally, the child was asked to explain how s/he knew which was which. Four possible total points could be obtained for this question, with each part worth one point.

Question 3. The child was shown three ten blocks and 12 units (|||............). The child was asked to write the number represented by the blocks. If the child's answer was correct (42), then the child was asked, "Do the four and the two have anything to do with how many blocks there were?" Each part of this question was worth two points. If a child could not write the correct number in part one, s/he would receive a zero for both parts of the question.

Question 4. Questions four was purposely structured with misleading cues to determine the strength of the children's place-value understanding. Three cups were placed on the table and the child was given 13 unit blocks with the following directions: "Place four blocks into each cup and tell me the total number shown by the blocks." This resulted in four blocks in each of the three cups with one extra block on the table. If the child did not give the correct number, the examiner asked him or her to recount the blocks. This procedure usually helped the child find the mistake. The child was then shown a card with the number 13 on it and asked whether the one had anything to do with how many blocks there were. Finally, the child was asked whether the three had anything to do with the blocks. Four points were possible for this question, two points for identifying the number as 13 and two points for being able to explain the relation between the digits and the blocks.

<u>Question 5</u>. The child was given 26 blocks and was told to put five in each of the five cups. The child was again asked to give the total number shown by the blocks. If the first answer was incorrect, the child was asked to count again. A correct answer was given a score of two. A card with the number 26 on it was shown to the child and the child was again asked to explain the relation between the two, the six, and the blocks on the table. Two points were awarded for a correct explanation. Since four points could be given for each of the five place-value understanding questions, a student could attain a maximum total score of 20 on this part of the assessment.

Experimental Group Procedures

The teachers in the Experimental Group followed the recording strip approach as outlined in Neufeld and Lucas' (1976) book, *Number Blox: A manipulative mathematics program, book A*. The total instruction period spanned 12 weeks. The lessons were scheduled two times a week and proceeded for about 30 minutes each day depending on unavoidable schedule changes.

The recording strip approach involves a gradual shift from activities that involve only the concrete materials to activities that use the materials and require symbol manipulation as well. In weeks one, two, and three of February, the children were given free exploration time in order to explore the nature of the Base 10 blocks. During this exploration time, the teachers would ask probing questions, when appropriate, to enable children to make observations concerning the size differences of the blocks, as well as to determine things like how many units blocks they could fit on top of a tens block.

In week four, trading games were played by all students. Students practiced trading 10 units blocks for one tens block. These games helped children to learn the ten-for-one trade which is an important step in the development of place-value understanding. In week five, race games to 100 (and higher) were played in which one child was the banker and the other two took turns rolling a die and counting out the correct number of blocks (remembering to trade ten ones for a ten). The child who first reached the total of 100 was the winner. As this game was played, children were being exposed to the concept of regrouping.

During the sixth week of instruction, the children were shown how to represent numbers with the Base 10 blocks and how to record numbers on strips. The teachers instructed the class as a whole on the recording strip procedure. When necessary, they reviewed the procedure with individual children as needed. The bulk of instructional time during this period was allotted to independent work on individual strips. Periodically, the teacher would assess each child's progress and check for understanding. For example, she would circle the numbers 23 and 32 and ask if the numbers were different since both numbers had a 2 and a 3 in them. If they said "No," she knew they did not yet grasp the concepts of place value. If they said "Yes," she knew that the students realized that the value of the digits in the number changed when the position of the digit changed.

Control Group Procedures

The Control Group also received place-value instruction approximately twice a week, but used *Invitation to mathematics*, a standard mathematics textbook published by Scott, Foresman (1988). After directed teaching of place-value concepts, one teacher generally assigned children related workbook pages to do and occasionally had them do counting activities with concrete materials such as connecting cubes and counters. The other Control Group teacher used workbook pages as well as a variety of seasonal activities such as the 100th day celebration and other calendar tasks. For these counting tasks, a variety of manipulatives were used as supplemental aids. Part of the 100th day activities required that students physically count their collections of 100 objects. In daily calendar activities, straws were counted and put into separate boxes marked ones, tens, and hundreds, thereby showing the number of days in school. It is important to note that no recording strips were used in either of the control group classrooms. Base 10 blocks, however, were available and were used at least once by some of the children in the Control Group.

Posttest

The posttest was administered after the twelve-week instruction period. On Part One, the children were asked to build the same numbers they had built on the pretest. The posttest repeated Parts One and Two of the pretest and had an additional section to measure the children's performance on standard addition problems that involved regrouping. This section was developed by the examiner from standard first-grade assessments of proficiency with the addition algorithms. It was not expected, however, that differences would occur between the Experimental and Control Group subjects because previous research (Lindquist, 1989) suggests that algorithms are learned and performed by rote at this age level. All 52 students involved in the study were given the posttest, and the scoring was the same as that described for the pretest.

Analysis

Means and standard deviations were computed for all pre- and posttest scores. Independent means *t*-tests were used to determine the significance of any differences between mean scores for experimental and control subjects. Chi-square analyses were also used where appropriate.

CHAPTER IV

Results

Introduction

The purpose of this study was to compare two approaches to place-value instruction in first grade in order to determine which was more effective in developing understanding of the concept. One approach used traditional textbooks and manipulative materials only as supplements. The second approach used place-value recording strips with manipulatives (specifically Base 10 blocks) as an integral part of the learning experience.

Four first-grade classes participated in the study. Children from two classes formed the Experimental Group, and children from the other two classrooms made up the Control Group. The Control Group was involved in traditional textbook-oriented approaches to place-value instruction. The Experimental Group experienced the Base 10 blocks and recording strip approach. Due to time constraints, a total of 52 students--13 from each classroom--were randomly selected for the pre- and posttest assessments. Part One of the pre- and posttests contained items which measured students' ability to construct correct representations of given numbers using Base 10 blocks. Part Two of both tests measured students' understanding of place value based on answers to five place-value questions. The posttest included an additional section as a measure of the children's performance on standard addition algorithms involving regrouping.

Pretest Results

On Part One of the pretest, the children were asked to build representations of three different numbers. Each student was shown, one at a time, three cards with a different number on each card. For each card, the student was asked to build two different representations of the number using the Base 10 blocks. The initial scoring classified each representation as belonging to one of four different categories: (1) no try or incorrect, (2) one-to-one collections, (3) noncanonical base ten, or (4) canonical base ten. Table 2 shows the results of this scoring; that is, the numbers and percentages of representations in each category for the two groups across Trial 1 and Trial 2 of the pretest. In examining Table 2, the Experimental Group built 116 correct representations while the Control Group built 125. Both groups, however, made the same number (52) of canonical representations. The Control Group showed a more marked preference for one-to-one collections on Trial 1, while the Experimental Group built a few more canonical representations. Overall, the two groups appear to be comparable in their abilities to build correct representations, with perhaps a slight advantage going to

Table 2

*Number and Percentage of Representations by Scoring Category for the Experimental and the Control Groups at Pretest**

	Trial 1		Trial 2		Total
	N	%	N	%	N
EXPERIMENTAL (N=26)					
No try or incorrect	9	12%	31	40%	40
One-to-one collections	32	41%	18	23%	50
Noncanonical base ten	3	4%	11	14%	14
Canonical base ten	34	43%	18	23%	52
Total	78	100%	78	100%	
CONTROL (N=26)					
No try or incorrect	2	2%	31	40%	33
One-to-one collections	48	62%	14	18%	62
Noncanonical base ten	4	5%	7	9%	11
Canonical base ten	24	31%	26	33%	52
Total	78	100%	78	100%	

*Please note: Total number of representations sums to 78 in each case because each of the 26 subjects built three numbers.

the Control Group.

An alternative scoring of Part One was designed to assess the flexibility and depth of the children's understanding. If a student could not make a correct representation or could make only one correct representation, then a zero was given for that number card. If the student had sufficient understanding to be able to build the number correctly in two different ways, then one point was given for that number card, producing a possible score range over the three number cards of zero to three. Table 3 displays the results of this analysis for both the Experimental and Control Groups at the pretest.

Table 3

Pretest Results for Experimental and Control Subjects on Part One Using Alternate Scoring

	Number of Children	
Score	Experimental	Control
0	6	4
1	5	9
2	5	1
3	10	12
Average score	1.73	1.81
Standard Deviation	(1.21)	(1.20)
Total instances of two correct representations	45	47

The pretest results displayed in Table 3 show that both the Experimental and Control Groups performed in essentially the same way. Even though the average score for the Control Group was slightly higher than that for the Experimental Group, the difference is not significant and the groups can be considered as essentially equal at the pretest.

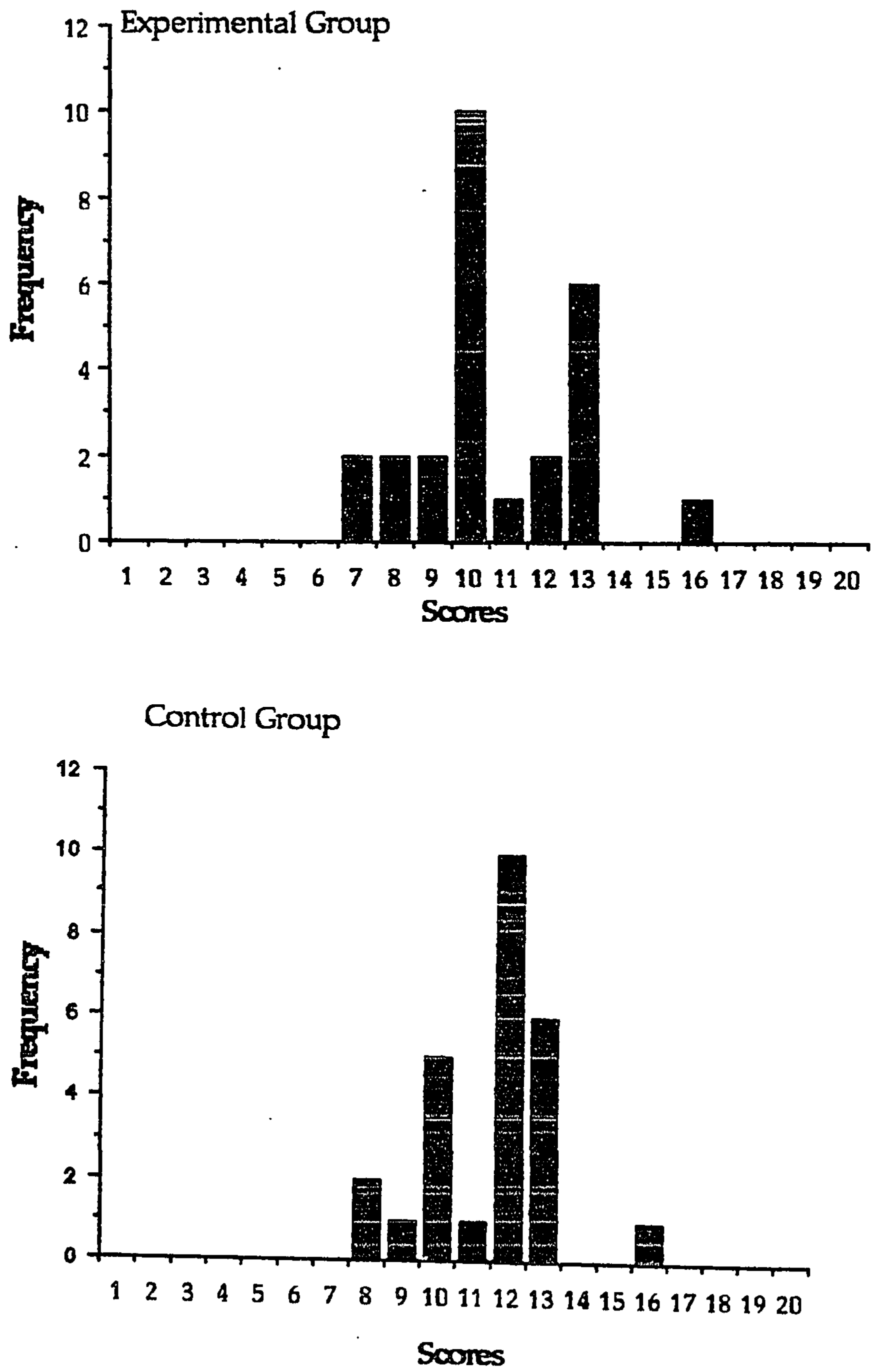
The histograms in Figure 1 are frequency distributions for Part Two (the five place-value questions) of the pretest. Each of the five questions was scored from zero to four. For all five questions, then, a maximum of 20 points could be obtained. The Experimental Group scores ranged from 7 to 16 with a mean of 10.65. The Control Group scores ranged from 8 to 16 with a mean of 11.53. As Table 4 shows, a t-test was performed on these data to determine if any significant difference existed between the Experimental and Control Groups on place-value understanding at the time of the pretest. The difference between the Experimental and Control means was .88. This difference produced a t value of .279, indicating a non-significant difference. The two groups can, therefore, be considered essentially equal at the start of the treatment.

Posttest Results

On Part One of the posttest (as in Part One of the pretest), each child was asked to build each of three different numbers two times. Across all 26 children in each group, there were, therefore, 78 possible constructions. Table 5 shows the numbers and percentages of representations in each scoring category for the two groups across Trial 1 and Trial 2 of the posttest.

In examining Table 5, it can be seen that on the posttest the Experimental Group had fewer no try or

Figure 1. *Histogram for Experimental and Control Group Scores on Pretest Part Two.*



<u>Note</u>. Maximum possible score for Pretest Part Two was 20.

Table 4

*Means, Standard Deviations and t-test Results for Part Two of the Pretest**

	Experimental (N=26)	Control (N=26)	*t*
Mean	10.65	11.53	
Standard Deviation	(2.16)	(1.74)	.279

*Please note: Maximum possible score was 20.

Table 5

*Number and Percentage of Representations by Scoring Category for the Experimental and Control Groups at Posttest**

	Trial 1		Trial 2		Total
	N	%	N	%	N
EXPERIMENTAL (N=26)					
No try or incorrect	0	0%	20	26%	20
One-to-one collections	21	27%	40	51%	61
Noncanonical base ten	0	0%	10	13%	10
Canonical base ten	57	73%	8	9%	65
Total	78	100%	78	100%	
CONTROL (N=26)					
No try or incorrect	3	4%	34	44%	37
One-to-one collections	33	42%	27	35%	60
Noncanonical base ten	4	5%	5	6%	9
Canonical base ten	38	49%	12	15%	50
Total	78	100%	78	100%	

*Please note: Total number of representations sums to 78 in each case because each of the 26 subjects built three numbers.

incorrect constructions than did the Control Group (only 20 for the Experimental Group, and 37 for the Control Group). Both groups built about the same number of one-to-one collections and noncanonical base ten representations. The Experimental Group, however, made considerably more canonical base ten representations than the Control Group (65 for the Experimental Group and 50 for the Control Group). These results confirm the expectation that children in the Experimental Group would make more canonical base ten representations because their treatment involved more experience building such representations with the Base 10 blocks.

Table 6 shows results from the alternative scoring of Part One, that is, the total number of instances on which two correct representations of a single number were made by Experimental and Control Group children on the posttest. If the student could not make a correct representation or could only make one correct representation, then a zero was given for that number card. If the student had sufficient depth of understanding to be able to build the number in two different ways, then one point was given for that number card making three the total possible.

As was the case in Table 5, the data in Table 6 show the Experimental Group's scores were numerically higher on this section of the posttest than were the Control Group's. The Experimental Group showed 58 instances of two correct

Table 6

Posttest Results for Experimental and Control Subjects on Part One Using Alternate Scoring

	Number of Children	
Score	Experimental	Control
0	4	9
1	2	2
2	4	4
3	16	11
Average score	2.23	1.65
Standard Deviation	(1.14)	(1.35)
Total instances of two correct representations	58	43

representations (a gain of 13 over their pretest performance). The Control Group showed 43 instances of two correct representations (a loss of 4 since the pretest). These data were examined using a Chi square analysis (Table 7). The χ^2 of 3.33 did not quite reach the 3.8 required for statistical significance at the .05 level.

The histograms in Figure 2 are frequency distributions for Part Two (the five place-value questions) of the posttest. The Experimental Group scores range from 10 to 18 with a mean of 13.73. The Control Group scores range

Table 7

*Comparison of Number of Instances on Which Two Correct Representations Were Made**

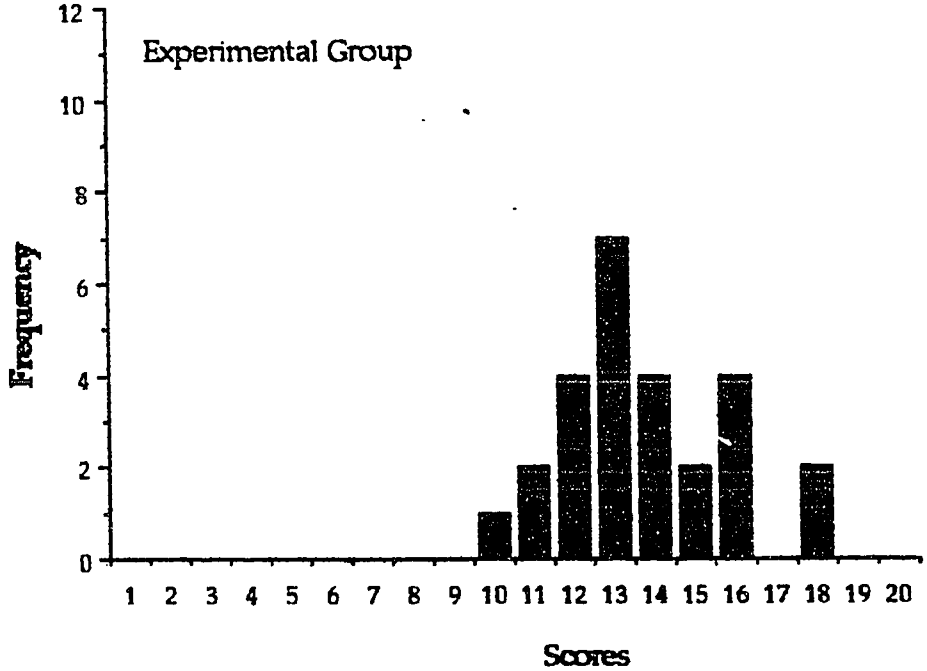
	Experimental (N=26)	Control (N=26)	χ^2
Pretest	45	47	
Posttest	58	43	3.33
Pre-Post Gain	13	- 4	

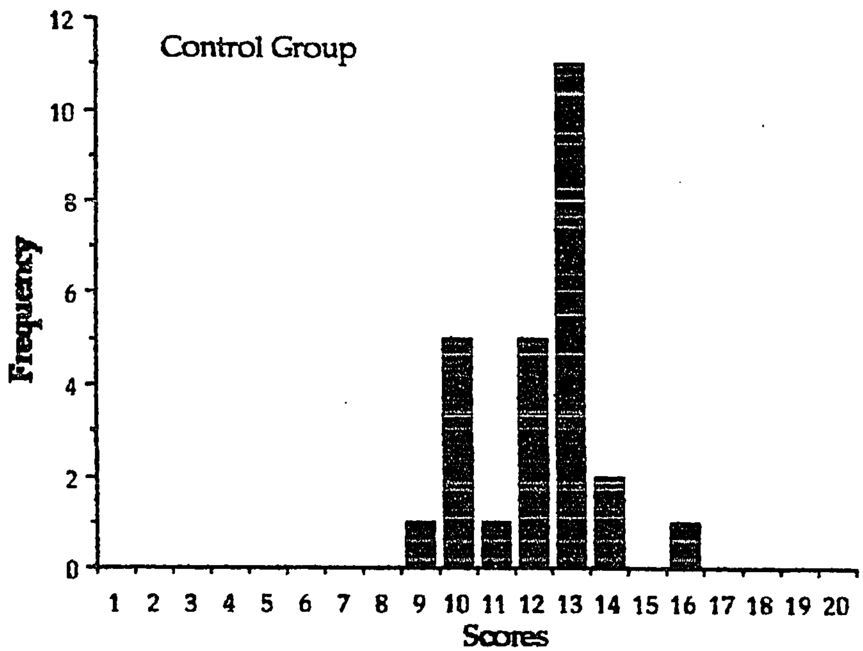
*Out of 78 possible.

from 9 to 16 with a mean of 12.19. As Table 8 shows, a *t*-test was performed on these data to determine if any significant difference existed between the Experimental and Control Groups on place-value understanding at the posttest. The difference between the Experimental and Control Group means was 1.54. Although the mean for the Experimental Group on the posttest was higher than that for the Control Group, the *t* value of .415 was not statistically significant. Table 9 shows the pre- to posttest differences in mean scores for both groups. The numerical gain for the Experimental Group from pre- to posttest on Part Two was 3.08. The numerical gain for the Control Group from pre- to posttest on Part Two was .66.

Four addition problems, which were developed by the examiner from standard first-grade assessments, were given

Figure 2. *Histogram for Experimental and Control Group Scores on Posttest Part Two.*





Note. Maximum possible score for Posttest Part Two was 20.

Table 8

*Means, Standard Deviations and t-test Results for Part Two of the Posttest**

	Experimental (N=26)	Control (N=26)	*t*
Mean	13.73	12.19	.415
Standard Deviation	(2.01)	(1.52)	

*Please note: Maximum possible score was 20.

Table 9

Comparison of Pretest and Posttest Scores Part Two

	Experimental N=26	Control N=26
Pretest Mean	10.65	11.53
Standard Deviation	(2.16)	(1.74)
Posttest Mean	13.73	12.19
Standard Deviation	(2.01)	(1.52)
Pre/Post Difference	3.08	.66

to both groups of students at the posttest. The intent was to measure each student's performance on the standard addition algorithm when regrouping was involved. Each correct execution of the algorithm was given one point. A student's score could range from zero, for no problems correct, to a maximum of four, for all four correct.

Means, standard deviations, and *t*-values were computed for the scores from the four problems and appear in Table 10. The mean for the Experimental Group was 1.92. The mean for the Control Group was 2.30. A *t*-test calculated from these data showed no statistical significance.

Table 10

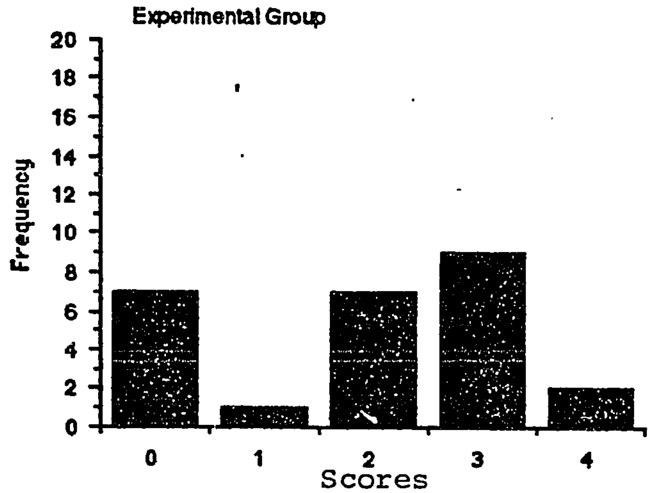
*Means, Standard Deviations and t-test Results for Algorithms on the Posttest**

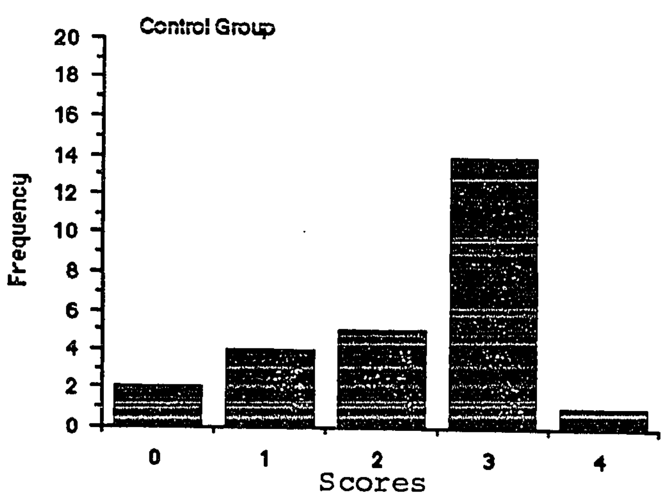
	Experimental (N=26)	Control (N=26)	*t*
Mean	1.92	2.30	
Standard Deviation	.93	1.02	.715

*Maximum possible score was 4.

The histograms in Figure 3 present the Algorithm Test data in graphical form. Both Experimental Group scores range from zero to four. It is important to note that although the Control Group mean is slightly higher than the

Figure 3. *Histogram for Algorithm Scores by Experimental and Control Groups on Posttest.*





<u>Note</u>. Maximum possible score was 4.

Experimental Group mean, the difference is small and not statistically significant. This is consistent with the hypothesis that no differences would be seen on the algorithms because previous research has demonstrated that many children are capable of rotely performing algorithms long before they understand the underlying place-value concepts.

Summary of Results

The results of this study indicated that, at the time of the pretest, children in the Experimental Group and the Control Group were comparable in their performance on measures of their ability to construct correct concrete representations of number and of their place-value understanding. On the posttest, children in the Experimental Group had higher scores on both measures, although the differences were not statistically significant. As predicted, no significant differences were found in the two groups' performance on a test of proficiency with the standard addition algorithm.

CHAPTER V

Summary and Conclusions

The problems most U.S. children experience with understanding place during their primary school years might be related to developmental level, as suggested by Ross (1986), or they might be related to the type of instructional strategies used to teach place-value concepts. Fuson and Briars (1990) concluded that the manner in which place value is introduced by many mathematics textbooks has led children to learn computation procedures by rote, instead of to understanding of the concepts.

The purpose of this study was to compare two approaches to place-value instruction in first grade to determine which was more effective in developing understanding of the concepts. One approach used a standard textbook occasionally supplemented by manipulative activities involving connecting cubes, straws, and, on at least one occasion, Base 10 blocks. The second approach used place-value recording strips and manipulatives (specifically, Base 10 blocks) as the primary means of instruction. The recording strip strategy developed by Neufeld and Lucas (1976) has been suggested in previous research as an alternative to textbook instruction. This approach was designed to lead children to develop a more complete understanding of place-value concepts through the use of manipulative materials, counting activities, and ample practice constructing concrete representations of numbers.

The study involved 52 first-grade students in four classrooms in a low- to middle-class California school. The analyses were carried out by comparing Treatment and Control Group performance on a series of both pre- and posttests designed to measure place-value understanding. On Part One of the pretest, the Control Group made slightly more correct constructions than the Experimental Group, however, the difference was not significant. The groups were therefore assumed to be equivalent on their ability to represent numbers correctly at the time of the pretest. A *t*-test for the difference between pretest mean scores on Part Two of the procedure, designed to assess place-value understanding, also produced a non-significant difference. The non-randomly assigned Experimental and Control Groups were therefore assumed to be approximately equal at the start of the treatment.

Posttest comparisons of Experimental and Control Group abilities to construct correct number representations and of understanding of place-value concepts showed no statistically significant differences. The Experimental Group, however, showed greater gains from pre- to posttest on both measures than did the Control Group. On Part One, the number representation tasks, the Experimental Group produced 136 correct constructions while the Control Group produced only 119. On Part Two, the place-value understanding tasks, the Experimental Group mean showed a gain of 3.08 from pre- to posttest. The Control Group gained only .66 from pre- to posttest.

The statistical evidence does not support the initial hypothesis that first-grade students who receive place-value instruction via the recording strip approach will achieve higher scores in place-value understanding than first-grade students who receive place-value instruction through the standard district approach. A number of plausible explanations concerning why the Experimental Group did not do significantly better than the Control Group can be considered. One reason might be the students' inability to transfer knowledge gained from using manipulative materials to paper and pencil tasks such as those on the assessments. An equally possible reason might be that the recording strip method is actually not effective in teaching place-value concepts to first-grade students. A third reason might be that the treatments were contaminated; that is, the Control Group children might have had sufficient exposure to the Base 10 blocks to perform as well as the Experimental Group children. Small sample size may have been a factor, as well as insufficiently sensitive measurement instruments. Any one, or a combination of these explanations, could have resulted in the non-significant differences.

Discussion

For the purposes of this study, it was assumed that a child could be said to understand place value when his/her responses on Part Two of the testing procedures for place-value understanding (adapted from Miura and Okamoto (1989)) revealed knowledge of the meanings of individual digits in a number, as well as the value of the digits in different positions in that number. A sample of comments from Experimental Group children during the assessment procedures may help to explain some of their numerical gains from the pre- to the posttest.

On Question #2, for example, which involved construction of the number 44, a girl in the Experimental Group was asked at the pretest stage to explain the meaning of each four in the number and simply shrugged. Twelve weeks later on the posttest she said, "Because the four is tens and the other four is ones." On Question #3, looking at a representation of the number 42 built with three tens and 12 ones, a boy in the Experimental Group could only explain at the pretest stage that, "The two is two ones and the four is four ones." On the posttest, when asked if the four and the two have anything to do with the number, he explained, "The 3 tens and 1 ten mean the 4 and the 2 is the 2 ones left." On Question #4, while considering a collection of 13 units blocks, four in each of three cups and one on the table, a child in the Experimental Group could only respond at the pretest, "one one, four cups". On the posttest, when asked to relate the blocks to the three in the number 13, the child explained, "There's 10 ones and three ones." Finally, on Question #5, which involved 26 units blocks, five in each of five cups and one on the table, a child in the Experimental Group could not answer on the pretest either interpretive item about the two and the six in the number. On the posttest, however, the child verbalized, "Yes, build two tens and six ones."

One explanation for the effectiveness of the recording strip approach in helping these children verbalize place-value concepts might be the greater amount of time spent using manipulatives, especially Base 10 blocks, as an integral part of the instruction. More time is spent in free exploration of number constructions which may lead to important discoveries of place-value concepts. In addition, a great deal of time is devoted to trading games in which children internalize the idea of trading 10 units blocks for one tens block, 10 tens for one hundreds block, and 10 hundreds for one thousand. Through race games, the children practice the concept of regrouping. As children are actively involved with the Base 10 blocks, the concepts of place value become more familiar to them. They are involved with multiple applications of newly acquired knowledge and are usually able to see the relevance of place value to dealing with real world quantities.

Children in the Control Group, who spent less time with manipulatives, experienced less active involvement in their place-value instruction. During whole group instruction, these children were not usually physically involved with any manipulatives, but were required to sit in their seats while the teacher demonstrated concepts on the board or manipulated straws herself, putting them into place-value boxes marked hundreds, tens, and ones in order to show the number of days the children had been in school. Workbook pages used with the Control Group children suggested the use of manipulatives, but did not require them. Children completed the exercises abstractly or by counting objects in pictures.

Recommendations

Although inconclusive because of non-significant differences, the results of this study were in the predicted direction and suggest the need for further research. A future study might well include three groups receiving three different treatments to distinguish between the benefits of Base 10 blocks alone and Base 10 blocks used with recording strips. One group would use recording strips and Base 10 blocks, a second group would use Base 10 blocks and no strips, and the Control Group would use the standard textbook approach.

Other specific recommendations concerning replications or extensions of this study in the future are:

1. Teachers who deliver the treatment should receive more training in the recording strip approach in order to ensure consistent implementation of the treatment across all classrooms.

2. More mathematics time should be scheduled into each day to ensure consistent exposure and reasonable continuity. Adequate time for such activities as trading games, race games, and building recording strips is crucial to the development of understanding.

3. Teachers need to have enough Base 10 blocks in the classroom so that all children may participate in the place-value activities together.

4. More time is needed for the researcher to observe both the control classrooms and the experimental classrooms to ensure the treatment is adequately implemented and that the control classes, in fact, use solely the traditional approach.

5. Measurement procedures need to be developed that do not involve the use of the experimental materials (Base 10 blocks). Materials such as beans, cars, and wheels could be used so that the assessment is more clearly of place-value understanding rather than of familiarity with the materials.

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APPENDICES

Appendix A

Pretest and Posttest Part One

Card 1

	Pretest	Posttest
Card (with the number 11 written on it)		
Read aloud the number on the card.	________	________
Show the number using the blocks. (teacher circles the answer)	|.	|. ....
No answer	________	________
Is there another way?	________	________

Card 2

	Pretest	Posttest
Card (with the number 13 written on it)		
Read aloud the number on the card.	________	________
Show the number using the blocks. (teacher circles the answer)	|...	|... ....
No answer	________	________
Is there another way?	________	________

Card 3

Card (with the number 30 written on it) Pretest Posttest

Read aloud the number on the card. ______________ ______________

Show the number using the blocks. (teacher circles the answer)

||| |||

||.......... ||..........

|.......... |..........
....
....

....
....
....

No answer ______________ ______________

Is there another way? ______________ ______________

Card 4

Card (with the number 28 written on it) Pretest Posttest

Read aloud the number on the card. ______________ ______________

Show the number using the blocks. (teacher circles the answer)

||........ ||........

|.......... |..........
....

....
....
....

No answer ______________ ______________

Is there another way? ______________ ______________

Card 5

Card (with the number 42 written on it)	Pretest	Posttest
Read aloud the number on the card.	________________	________________
Show the number using the blocks. (teacher circles the answer)	||||..	||||..
	|||.......... ..	|||.......... ..
	||...........	||........... ....
	....	....
No answer	________________	________________
Is there another way?	________________	________________

Appendix B

Pretest and Posttest Part Two

<u>Question #1</u> - Card with 32 written on it

	Pretest	Posttest
Teacher: "Point to the numeral in the ones position."	________	________
"Point to the numeral in the tens position."	________	________
Teacher: "Use the blocks to show the 3."	________	________
Teacher: "Use the blocks to show the 2."	________	________

<u>Question #2</u>

	Pretest	Posttest
Teacher makes the number 44 using three tens blocks and 12 ones blocks.		

||||....

Teacher asks:

"What number do the blocks make?"	________	________

Teacher shows a card with 44 written on it.

Teacher asks:

"Which set of four blocks represents the four in the tens position?"	________	________
"Which set of four blocks represents the four in the ones position?"	________	________
"Explain how you know which one was which?"	________ ________ ________	________ ________ ________

Question #3

	Pretest	Posttest
Teacher makes the number 42 using three tens blocks and 12 ones blocks.		
|||............		
Teacher asks:		
"Write the number built?"	______	______
"Do the four and the two have anything to do with how many blocks there were?"	______ ______ ______	______ ______ ______

Question #4

	Pretest	Posttest
Teacher gives the child 13 unit blocks.		
....		
Teacher gives the following directions:		
"Put four blocks each into cups."		
"Give me the total number shown by the blocks?" (13)	______	______
Teacher shows card with a 13 written on it.		
Teacher asks:		
"Do the one and three have anything to do with how many blocks there were?"	______ ______ ______	______ ______ ______

Question #5

	Pretest	Posttest
Teacher gives the child 26 unit blocks.		
....		
Teacher gives the following directions:		
"Put five blocks each into cups."		
"Give me the total number shown by the blocks?" (26)	____	____
Teacher shows card with a 26 written on it.		
Teacher asks:		
"Do the two and the six have anything to do with how many blocks there were?"	____ ____ ____	____ ____ ____

Appendix C

Posttest Algorithms

$$\begin{array}{r} 8 \\ +\ \underline{7} \end{array} \qquad \begin{array}{r} 12 \\ +\ \underline{9} \end{array}$$

$$\begin{array}{r} 24 \\ +\ \underline{18} \end{array} \qquad \begin{array}{r} 153 \\ +\ \underline{67} \end{array}$$